Price Discrimination with Endogenous Participation*

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Abstract

To what extent should platforms enable price discrimination by sellers when buyers can vote with their feet? I assume that buyers are endowed with an outside option and a belief about their valuation before they decide whether to participate in the platform or not. For the platform, endogenous participation creates a trade-off between surplus extraction and participation. It also imposes a constraint: when participation increases with valuations, the segmentation rules that deliver the highest buyer surplus under full participation cannot be implemented. I characterize the feasible welfare frontier and derive the platform's optimal segmentation under three alternative assumptions about buyer information. Overall, while endogenous participation can push platforms to give more surplus to buyers, it can also limit their ability to implement the most buyer-friendly segmentation rules.

Keywords: Price Discrimination, Information Design, Digital Platforms

JEL classification: D82, D83, L86, M31

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1 Introduction

In digital markets, platforms use buyer data to segment buyers and inform seller pricing. By acting as information intermediaries, these platforms enable sellers to engage in price discrimination. This allows seller-aligned platforms to increase seller margins and overall profits. For instance, marketplaces like Amazon use segmentation rules to allow sellers to run targeted promotions within their platforms[1], while advertising platforms such as Google and Meta enable price discrimination through audience-targeted campaigns[2]. At the same time, buyers are not passive recipients of these policies. They can often respond strategically by choosing not to create accounts or by reducing their activity if they perceive the platform's information policy as too extractive[3].

This paper studies how much price discrimination a platform should enable when buyers can choose to walk away. Endogenous participation creates a trade-off: providing sellers with finer buyer segments increases seller margins per participant but reduces buyer participation.

The model features a monopolistic seller offering one good and a unit mass of buyers interacting through a platform. The platform publicly commits to a segmentation rule that determines how buyers are split into segments. Each buyer is endowed with a belief about their valuation and an outside option. Buyers form rational expectations about the prices they will face, given the platform's announced segmentation. They then decide whether to participate, choosing to join only if their expected surplus exceeds their outside option. Once buyers decide to participate, their valuations are realized and observed both by themselves and the platform. Participants are assigned to segments according to the platform's rule. Finally, the seller observes the composition of each segment and sets one price per segment. This timing implies that participation decisions are managed at the platform level. This means sellers do not internalize how their pricing affects buyer participation. This reflects online

[1]See Amazon Brand Tailred promotions, last access on October 2025

[2]See Meta Ads Manager promo codes, and Google Promotion Assets, last access on October 2025. For literature about the interplay between targetted advertising and price discrimination, see Iyer et al. (2005) and Esteves and Resende (2016)

[3]See Hippel and Hillenbrand (2025)

marketplaces where sellers are short-lived or fragmented and thus lack credible commitment to prices that could influence overall participation.

The trade-off generated by endogenous participation incentivizes the platform to implement surplus splits more favorable to buyers. If buyer participation were guaranteed, a seller-aligned platform would design segments to maximize extraction. To do this, it creates a separate segment for each valuation, fully informing the seller and allowing them to extract the entire market surplus, leaving buyers with zero surplus. However, this strategy can become sub-optimal if buyers can walk away. Anticipating zero surplus, a large share of buyers refuses to join the platform, which causes realized profits to decrease. To induce participation, the platform must share surplus with buyers. Bergemann et al. (2015) characterize the set of welfare splits between buyer surplus and seller profits that are implementable by some segmentation. They show that the platform can generate buyer surplus by exploiting pooling externalities.[4] When the platform pools low- and high-valuation buyers in the same segment, low-valuation buyers exert a positive externality on high-valuation buyers by inducing the seller to set lower prices.

The mechanism driving the buyer participation side of the trade-off is the buyer's expectation of surplus, which is shaped by their private belief about their own valuation. Because beliefs can differ across buyers, this leads to differential participation: buyers enter the market at different rates depending on their valuation, which endogenously alters the market's composition. To unpack how this selection affects the platform's choice, this paper considers three informational regimes. First, I consider a setting where buyers are uninformed about their valuation at the participation stage. Since they all share the same belief, participation decisions are uncorrelated with valuations. This shuts down differential participation, and isolates the participation-extraction trade-off, as the market changes in size but not composition. Second, I consider a setting where buyers observe their valuation before the participation stage. Here, participation is maximally correlated with valuations, which constrains the

[4]See Galperti et al. (2024)

platform's ability to generate buyer surplus through pooling. To maintain tractability, I analyze this case in a two-valuation setting. Finally, to bridge these two extremes, I introduce a partial information structure called bottom-weight beliefs, where beliefs differ only in the probability of having the lowest valuation. This tractable framework allows me to characterize the welfare frontier and to analyze how the degree of buyer information shapes the set of implementable surplus splits.

The first main finding is that *when buyers have information about their valuation, the platform cannot implement the most buyer-friendly segmentation rules.* This is because surplus creation relies on pooling externalities: low-valuation buyers reduce prices within pooled segments, generating surplus for high-valuation buyers. High-valuation buyers expect this benefit and participate. Meanwhile, low-valuation buyers recognize their role as "externality providers" and expect less surplus, so they participate less. This adverse selection problem creates a market skewed toward high-valuation buyers, which limits the platform's ability to generate surplus through pooling.

The second main finding is that *forces often considered pro-consumer, such as greater buyer information or more elastic participation, have ambiguous effects on buyer surplus.* Platforms are incentivized by endogenous participation to implement more buyer-friendly surplus splits, to reveal less buyer information by implementing coarser segmentation rules that pool buyers of different valuations to generate buyer surplus. This means that if the platform is not implementing the buyer-optimal segmentation, more elastic buyer participation increases surplus among participating buyers. But this incentive meets a limit at the buyer-optimal segmentation. Beyond this point, the platform cannot expand buyer surplus. The analysis reveals that the buyer-optimal segmentation moves in the opposite direction. As participation becomes more elastic to expected surplus, or, as the partial information analysis reveals, as buyers become better informed, the surplus generated by the buyer-optimal segmentation decreases. In this case, pro-consumer forces decrease surplus among participating buyers.

The remainder of the paper is organized as follows. After discussing related work, Section 2 presents the model and some preliminary results – namely, that the equilibrium segmentation is always efficient in the sense that all participating buyers are served, and that the platform's choice of segmentation can be reduced to selecting a family of expected surplus profiles per belief group that is implementable by some segmentation. Section 3 studies the platform's problem under no-information and full-information regimes. Finally, Section 4 addresses the problem under the bottom-weight belief structure.

Related work. Starting with Pigou (1920), the literature on third-degree price discrimination examines how consumer surplus, producer surplus, and total surplus vary when the market is segmented (Schmalensee, 1981; Varian, 1985;Aguirre et al., 2010). Traditionally, these market segmentation rules are taken as exogenous, with fixed distributions of willingness-to-pay within each segment. Bergemann et al. (2015) depart from this by making segmentation endogenous, framing it as an information design problem where the segments and their valuation distributions are an informed intermediary's choice. Building on this, recent research explores the effects of segmentation on competition (Elliott et al., 2024) and how pooling externalities determine the value of data (Galperti et al., 2024). Other contributions modify the intermediary's objective, such as Banerjee et al. (2024), who study intermediaries with fairness goals, and Augias et al. (2025), who examine redistributive price discrimination.

Closer to this work are papers that incorporate buyer-side agency through privacy or voluntary disclosure, which impose incentive compatibility constraints on buyers. Examples include Ichihashi (2020) and Hidir and Vellodi (2021). Ali et al. (2023) also model voluntary disclosure where buyers decide which valuation-revealing evidence to provide. In this case, when the buyers have rich evidence, buyer information revelation endogenously determines segment compositions. Gambato and Peitz (2025) study this framework applied to platform governance with endogenous participation. They show that seller-aligned platforms who can both choose the buyer disclosure regime and seller fees can have misaligned incentives with the seller due to endogenous buyer participation. Our paper adds to this literature by considering a setting where the information intermediary fully controls the segmentation, while buyers face an individual rationality constraint rather than incentive compatibility. That is, buyers do not have strategic disclosure incentives, but strategic participation incentives.

The economic forces driving our results also echo those in the literature on two-sided platforms. The classic insight shows that the platform pricing structure, which determines which side is subsidized, depends on participation elasticity and network effects (Rochet and Tirole (2003); Armstrong (2006)). Of particular relevance are works studying price discrimination in two-sided markets: de Cornière et al. (2025)) analyze discriminatory seller fees, showing that third-degree price discrimination on sellers can enhance participation and welfare; Montes et al. (2019) consider price discrimination with endogenous privacy choices. Our approach differs in that the platform works under a fixed fee structure and has no direct control over the retail price paid by consumers. Instead, it acts solely as an information intermediary with the power to design the information structure. This reflects many real-world platforms such as Google, Meta, or Amazon, which often have no control over the retail price but exert influence through audience segmentation that allows firms to offer personalized promotions. To the best of our knowledge, this is the first paper to study this setting.

Finally, our framework views segmentation as endogenously determined by an information design choice but incorporates endogenous buyer participation. This situates our work within the growing literature on information design with endogenous states, which includes research on test design under endogenous participation (Rosar, 2017), falsification (Perez-Richet and Skreta, 2022), investment (Augias and Perez-Richet, 2023; Zapechelnyuk (2020)), and delegation (Bizzotto et al., 2020). This paper contributes by studying how informing about a market can reshape its composition.

2 Model

The model studies a market where a monopolistic seller reaches buyers through an intermediary platform. The platform can leverage its knowledge of buyer valuations to design a segmentation rule, an information policy that enables the seller to price discriminate. Buyer participation is endogenous: buyers have outside options and may refuse to join the platform if they expect extractive pricing strategies.

2.1 Economic Environment

A monopolistic seller offers a single good at zero production cost to a unit mass of buyers with unit demand, who are heterogeneous in their valuation v for the good. The valuations are drawn from the finite ordered set $V := \{v_1, ..., v_K\}$, where $0 \leq v_1 < ... < v_K$. Buyer preferences are quasi-linear; a buyer with valuation v purchasing the good at price p receives surplus of $v - p$, and zero otherwise. Consequently, buyers purchase if and only if their valuation exceeds or equals the price. The platform's objective is to maximize seller profits, reflecting a business model where it earns a fixed share of seller revenue.

Markets. A *market* is a probability distribution $\mu \in \Delta(V)$ over the set of buyer valuations. Facing a market μ, the seller sets a single price p to maximize profits, $\pi(\mu) = \max_{p \in V} \left\{ p \cdot \sum_{k: v_k \geq p} \mu_k \right\}$. The optimal price is $p^\star(\mu)$, with ties broken towards the lowest price. The total per-capita surplus in the market is $w(\mu) = \sum_k v_k \, \mu_k$. The initial market, prior to any participation decision, is described by a common knowledge distribution μ^0. This generates a benchmark seller uniform price $p^0 = p^\star(\mu^0)$, uniform profits $\pi^0 = \pi(\mu^0)$, and total per-capita surplus in the initial market $w^0 = w(\mu^0)$.

Segmentation rules. The platform can influence market outcomes by informing the seller about buyers' valuations. It does so by committing to a *segmentation rule*, a mapping $\sigma : V \to \Delta(S)$ assigning each buyer valuation v_k to a segment $s \in S$ with probability $\sigma(s|v_k)$.

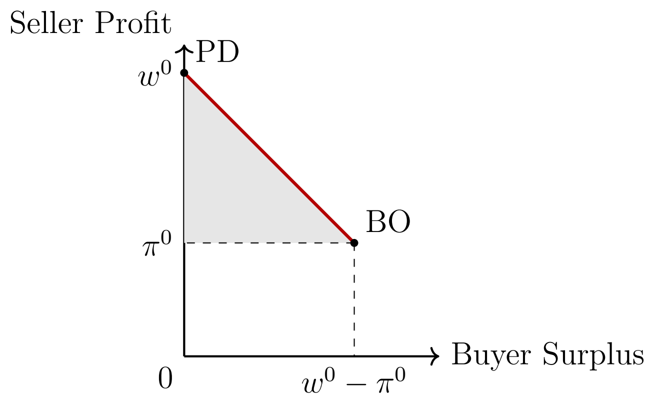


Figure 1: The welfare triangle, from Bergemann Brooks and Morris (2015)

Applying σ to a any market μ splits the buyer population into sub-markets, or *segments*. Each segment s has a market share $\tau_s = \sum_k \mu_k \, \sigma(s|v_k)$ and a buyer composition μ^s, where the share of v_k in segment s is $\mu_k^s = \frac{\sigma(s|v_k)\mu_k}{\tau_s}$, for $\tau_s > 0$. By construction, these segments must satisfy Bayes-plausibility, meaning that they average back to the initial market, $\sum_{s \in S} \tau_s \mu^s = \mu$.

Viewed through a pricing lens, the platform can choose these segmentation rules strategically to influence seller prices. This places the platform in the role of an *information designer* who commits to an experiment (the segmentation) that influences downstream actions (the prices). A useful simplification restricts attention to *direct segmentation rules*, where each segment s is labeled by the unique optimal price $p^\star(\mu^s)$ charged by the seller. This restriction is without loss of generality [5], and means that the platform's problem can be represented as partitioning the market according to the final seller prices.

Segmenting the market allows the platform to implement any welfare split in the shaded triangle of Figure 1.[6] The red segment corresponds to the Pareto frontier, the set of *efficient* segmentation rules. These rules maximize total surplus by never assigning buyers to segments where the posted price exceeds their valuations, ensuring all buyers are served.

Definition 1. A segmentation rule σ is efficient if $\sigma(p|v) = 0$ for all prices $p > v$

The Pareto frontier's endpoints are the seller-optimal and buyer-optimal outcomes, respec-

[5] See Bergemann and Morris (2019)
[6] See Bergemann et al. (2015)

tively. First, *Perfect Price Discrimination (PD)*: the platform reveals buyers' valuations fully, separating each into a pure segment. The seller charges each buyer their valuation, extracts all the surplus w^0 and leaves buyers with zero. Second, *Buyer-optimal (BO)* segmentation rules, constructed by maximally pooling buyers with different valuations. The pooling continues until the seller is indifferent between charging the lowest price in the segment and the uniform price p^0. This ensures that the seller obtains their reservation profits π^0, and maximizes buyers' surplus at $w^0 - \pi^0$.

Buyer information. The key departure from the standard models is that buyers endogenously make the choice of participating in the platform. To make this decision, buyers rely on two private components: an outside option and a belief about their valuations at the participation decision stage.

Each buyer has *private outside option* u, representing the surplus they obtain from not participating in the platform. This outside option is independently drawn from a continuous, log-concave distribution G supported on $\mathbb{R}_+$, and is independent of the buyers' valuation v.

Buyers decide to participate by comparing their expected surplus from the platform to their outside option. This expectation is formed based on their private belief $\beta \in \Delta(V)$ about their valuation, which may differ across buyers. Denote as $P(\beta)$ the share of buyers who hold belief β. Beliefs are Bayes-plausible, that is, they average back to the initial market $\sum_\beta P(\beta)\beta = \mu^0$.

2.2 The game and equilibrium

Timing. The game has four stages. First, the platform publicly commits to a segmentation rule σ, and each buyer is endowed with a private belief β and a private outside option u. Second, buyers form rational expectations about seller prices under σ, and decide whether to participate. The set of participating buyers forms an endogenous market. Third, these buyers' valuations are realized, and they are sorted into segments according to the segmentation rule σ. Finally, the seller observes the segment composition of each segment and sets the segment-specific optimal prices.

Buyer participation. A buyer with belief β expects a surplus under segmentation σ given by

$$\kappa(\sigma|\beta) = \sum_k \beta_k \sum_s \sigma(s|v_k)(v_k - p_s^e)^+,$$

where p_s^e is the price buyer anticipates in segment s and $(x)^+ = \max(x, 0)$. The buyer participates if this expected surplus exceeds the outside option u, that is $\kappa(\sigma|\beta) \geq u$. The participation rate for all buyers sharing belief β is therefore $e(\beta, \sigma) = G(\kappa(\sigma|\beta))$.

Realized market. Buyers with different beliefs participate at different rates. This can change both the size and composition of the market, and in turn, the segments that the seller will face. The total mass of participating buyers with valuation v_k is $\tilde{\mu}_k = \sum_\beta P(\beta)e(\beta, \sigma)\beta_k$.

Applying the segmentation rule σ to this participating market, the mass of buyers with valuation v_k assigned to segment s is

$$\tilde{\mu}_k^s = \sum_\beta P(\beta)\, e(\beta, \sigma)\, \beta_k\, \sigma(s|v_k)$$

.

Equilibrium. The solution concept is Perfect Bayesian Equilibrium. An equilibrium is a tuple (σ, e, p) such that: (i) the platform's segmentation σ maximizes its objective given the participation rates and prices; (ii) for each belief β, participation rate $e(\beta, \sigma)$ follows from optimal buyer participation decisions under rational expectations; and (iii) the seller price for each segment p_s is the optimal response to the resulting segment compositions $\tilde{\mu}^s$.

2.3 Efficiency and implementable surplus splits

Efficiency. It is useful to restrict attention to the class of *efficient* segmentation rules. The following lemma establishes that it is without loss of generality.

Lemma 1. *For any segmentation rule σ, there exists an efficient rule $\hat{\sigma}$ such that buyer surplus and participation remain unchanged, and platform profits are weakly higher.*

Buyers assigned to a segment priced above their valuation get zero surplus and generate no platform revenue. By reallocating these unserved buyers to segments priced at their valuation, the platform can increase profits without affecting equilibrium prices or buyer participation. The formal proof can be found in Appendix A. This result implies we can restrict the platform's choice to the Pareto frontier of Figure 1, as any efficient rule maximizes the total surplus generated by participating buyers.

Welfare. Under equilibrium conditions, the realized profits and buyer surplus can be directly derived from the buyer's expected surplus $\kappa(\sigma \mid \beta)$. Rational expectations require that the average realized surplus for buyer with belief β must equal their expected surplus $\kappa(\sigma|\beta)$. Since by Lemma 1 the segmentation is efficient, the average total surplus generated by buyers with belief β is their average valuation $\mathbb{E}[v|\beta]$. Therefore, the seller's profit per buyer with belief β is the residual, $\mathbb{E}[v|\beta] - \kappa(\sigma|\beta)$.

Summing these per-participant profits across belief groups weighted by their participation rates gives realized profits $\Pi(\sigma)$ and buyer surplus $K(\sigma)$:

$$\Pi(\sigma) = \sum_{\beta} P(\beta)\ e(\beta, \sigma)\ (\mathbb{E}[v|\beta] - \kappa(\sigma|\beta)), \qquad K(\sigma) = \sum_{\beta} P(\beta)\ e(\beta, \sigma)\ \kappa(\sigma|\beta)$$

This simplifies the platform's problem: rather than optimizing over the high-dimensional space of segmentation rules σ, the platform effectively chooses an expected surplus profile $\{\kappa(\sigma|\beta)\}_{\beta}$ across all profiles that are *implementable* by some segmentation. The platform's problem thus reduces to selecting the implementable profile that maximizes realized profits.

2.4 Discussion of key assumptions

This section discusses several key assumptions that define the scope of the analysis and clarify their interpretation.

Buyer's outside option. Buyers' outside options u are assumed to be independent of their valuations v. This means that these outside options are interpreted as participation costs, such as privacy concerns, inequity aversion, time, or platform access costs, distinct from valuation-driven substitution effects. These costs are sunk upon joining, which removes their influence on subsequent purchase decisions. This applies to settings where a general cost, rather than a specific competing option, determines participation.

Seller's Pricing and Commitment. The model places the management of buyer participation at the platform level. The seller acts myopically, setting prices only after observing the final composition of the participating market. This is a natural implication of the timing. Once buyers enter, their outside option is sunk, so a pre-announced low price is not credible at the pricing stage. However, a seller could commit through reputation or contracts. For instance, contracts with the platform could force price commitment and would be first-best for the platform. This model applies to marketplaces hosting short-lived, anonymous or fragmented sellers. In this case, they have no ability to either build a reputation or contract with the platform, which is common in online marketplaces. By contrast, the platform can commit. Large platforms are often long-lived, and face reputational and regulatory scrutiny. This often forces them to publish and commit to their information policies.

3 Trade-off and constraint: two benchmark cases

To build intuition, this section analyzes two limiting cases of buyer information. First, a setting with uninformed buyers neutralizes differential entry, allowing for a clean characterization of the platform's participation-extraction trade-off. Second, a setting with fully informed buyers, which, to maintain tractability, is analyzed in a two-type setting, introduces the strongest possible differential entry. It reveals the implementability constraint that buyer self-selection imposes on the platform's ability to create buyer surplus. Together, these stylized cases illuminate the core mechanisms at play before the analysis of the partial information environment.

3.1 Uninformed buyers: the participation-extraction trade-off

This section analyzes the platform's problem under the assumption that buyers are uninformed, meaning each buyer's belief about their valuation is the prior, $\beta = \mu^0$. Because all buyers share the same belief, they face the same expected surplus $\kappa(\sigma)$ under any segmentation rule σ. Consequently, participation decisions are uncorrelated with buyer valuations, and only depend on each buyer's outside option.

This lack of correlation results in *proportional participation*: buyers enter the market in the same relative proportions as in the initial market μ^0. Although the total market size can shrink, its composition will always remain identical to the initial market. From the seller's perspective, this means that the act of a buyer participating is uninformative of the buyer's valuation.

3.1.1 The set of implementable surplus splits

This proportional participation property shapes the platform's choice set, formalized in the following lemma.

Lemma 2. *With uninformed buyers, the set of average buyer surpluses implementable in an efficient equilibrium is the interval* $[0, w^0 - \pi^0]$

The proof, which can be found in Appendix B, relies on proportional participation: the participating market has the same composition as the initial market μ^0, differing only in size. Therefore, for any segmentation rule, the resulting average per-participant surplus κ matches the average per-capita surplus that the rule would yield under full participation.

This establishes an equivalence to the full-participation benchmark, where any per-capita buyer surplus in $[0, w^0 - \pi^0]$ is implementable by some segmentation [7]. Accordingly, the same set of per-participant surpluses is attainable here as well. Given Lemma 1, which restricts attention to efficient segmentation rules, any implementable outcome must be on the welfare frontier. This implies that for each such surplus κ, the per-participant profit is $\pi = w^0 - \kappa$.

3.1.2 The equilibrium surplus split

With the platform's choice set and the profit per participant established, we can now solve for equilibrium. The platform chooses an average buyer surplus κ to maximize realized profits, which equal the product of the participating mass of buyers $G(\kappa)$ and the per-participant profits $w^0 - \kappa$. Formally, the platform wants to

$$\max_{\kappa \in [0, w^0 - \pi^0]} G(\kappa) \cdot (w^0 - \kappa)$$

Proposition 1. *The equilibrium per-participant surplus,* $\kappa^\star$*, satisfies:*

1. *If an interior solution exists, it uniquely solves the first-order condition*

$$\frac{g(\kappa^\star)}{G(\kappa^\star)} = \frac{1}{w^0 - \kappa^\star}$$

2. *If no interior solution exists, the equilibrium is at the corner, with the platform choosing*

[7] See Theorem 1 of Bergemann et al. (2015)

the most buyer-friendly split possible, $\kappa^\star = w^0 - \pi^0$.

To unpack the intuition, multiply both sides of the first-order condition by $\kappa^\star$:

$$\kappa^\star \frac{g(\kappa^\star)}{G(\kappa^\star)} = \frac{\kappa^\star}{w^0 - \kappa^\star}$$

The left-hand side of this equation is the *participation elasticity* with respect to surplus, denoted $\epsilon_G(\kappa)$. It measures the percentage increase in participating buyers from a 1% increase in surplus offered. The right-hand side is the *surplus-to-profit ratio*, κ/π, representing the share of welfare allocated to buyers relative to the seller. Therefore, the platform raises κ until the percentage increase in participants matches the percentage fall in profit per participant. An illustration of this result can be found in Figure 2. The first-order condition ensures that the realized profits are maximized by choosing the $\kappa^\star$ at the tangency between the set of implementable average buyer surplus $\kappa \in [0, w^0 - \pi^0]$ and the highest iso-profit curve.

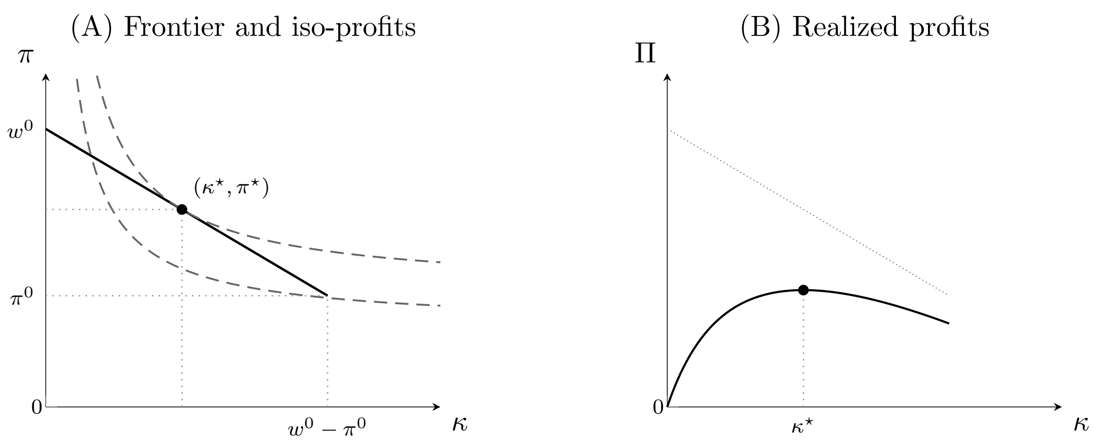


Figure 2: Uninformed buyers. Panel (A) shows the implementable frontier and two iso-profit loci; the optimum is the tangency. Panel (B) plots realized profits on the same scale. Parameters: $w^0 = 5$, $\pi^0 = 2$, $G(\kappa) = \kappa/(\kappa + 1)$.

This optimality condition allows us to analyze how the platform adapts its strategy to the market environment. A natural question is how the equilibrium surplus $\kappa^\star$ changes as *participation pressure* increases–that is, when buyers have better outside options, making their participation more sensitive to the surplus offered. I formalize this by comparing two outside option distributions, G_1 and G_2.

Assume G_2 dominates G_1 in the reverse hazard rate order, that is $\frac{g_2(x)}{G_2(x)} > \frac{g_1(x)}{G_1(x)}$ for all x. A higher reverse hazard rate implies a higher participation elasticity: under G_2, any increase in surplus yields a larger percentage increase in participation.

Proposition 2. *Let $\kappa^\star(G)$ denote the equilibrium buyer surplus for an outside option distribution G. If G_2 dominates G_1 in the reverse hazard rate order, then:*

$$\kappa^\star(G_2) \geq \kappa^\star(G_1)$$

This result directly follows from the first-order condition defining the equilibrium, and formalizes participation pressure. When buyers have better outside options, increasing the elasticity of participation, the platform's profit-maximizing response is to choose segmentation rules that give more surplus to the buyers. The platform internalizes how the segmentation impacts market size and thus limits price discrimination to sustain participation.

The uninformed-buyer case illustrates the platform's participation-extraction trade-off. To maximize extraction, the platform could reveal full information to the seller, enabling higher prices and margins, but risking buyers walking away from deals they perceive as too extractive. Conversely, maximizing participation entails pooling buyers to keep prices low but may leave profits on the table. The analysis shows that a profit-maximizing platform balances these forces. It strategically limits information revelation, choosing a segmentation that is not fully informative to keep the market attractive to buyers. This balance shifts when buyers' participation becomes more elastic, as formalized in Proposition 2. This pressure forces the platform to prioritize participation with more generous surplus splits.

3.2 Fully-informed buyers: the implementability constraint

This section analyzes the platform's problem assuming buyers know their valuation v at the participation stage. Unlike uninformed buyers who consider the average surplus, informed buyers consider valuation-specific expected surpluses $\kappa(\sigma|v)$. Because these surpluses differ across valuations, participation decisions become correlated with valuations, leading to *differential participation*. This endogenously alters not just market size but also its composition, making buyer participation informative to the seller.

First, informed buyers can cause market *unravelling* when $G(0) = 0$. The lowest-valuation buyers v_1 never face prices below v_1[8], yielding zero surplus. If no buyers participate with zero surplus $G(0) = 0$, all v_1 buyers do not participate. The effective market then becomes $\{v_2, ..., v_K\}$. The same logic then applies to the new lowest type, v_2, and iterates up the valuation set and collapses participation entirely. Second, analyzing the $K-$valuation case is challenging. Multiple segmentation rules can generate the same total expected surplus but differ in how they distribute this surplus across buyer valuations[9]. This creates a high-dimensional, non-linear problem that is not analytically tractable.

Given these points, I adopt two simplifying assumptions. First, a fraction $\lambda \in (0, 1)$ of all buyers are *captive* and always participate [10][11]. This ensures a non-zero baseline participation including v_1 buyers. Second, I focus on the two-valuation setting $V = \{v_1, v_2\}$, where μ is such that $p^0 = v_2$. This enables us to retain the core economic insights of the model, while obtaining a complete and tractable characterization of implementable segmentation rules and platform optima. This tractable setting provides insights that serve as a foundation for future work.

In a two-valuation market, each segment can only be priced at either v_1 or v_2. Therefore, any efficient segmentation rule divides buyers into exactly two segments: one priced at v_1, and one at v_2. To ensure efficiency, all buyers with valuation v_1 must be assigned to the v_1-priced segment. The platform's only choice is how to allocate the high valuation buyers between the two segments.

[8] In any market μ, pricing at $p < v_1$, raising to $p = v_1$ strictly increases revenue while still serving the whole market.

[9] see Augias et al. (2025)

[10] This means that the outside option distribution is of the form $G(x) = \lambda + (1 - \lambda)\tilde{G}(x)$, where $\tilde{G}(x)$ is a log-concave, continuous CDF on $\mathbb{R}_+$

[11] We allow for a mass at $u = 0$

This choice can be parametrized by $q \in [0, 1]$, representing the share of v_2 buyers placed in the v_2 segment. This segment perfectly reveals their valuation, enabling perfect price discrimination. The remaining fraction $1 - q$ is assigned to the v_1-segment, pooled with all v_1 buyers. Figure 3 illustrates these segmentation rules σ_q.

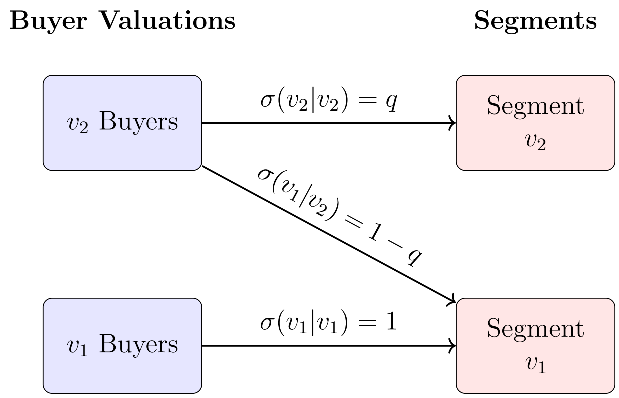


Figure 3: Representation of equilibrium segmentation rules

3.2.1 The set of implementable surplus splits

This parametrization captures the tension in surplus-creating segmentation rules. For any $q < 1$, the platform creates a pooled segment with the intention that it be priced at v_1. This is the *efficient price* for this segment, as it is the only price at which both low- and high-valuation buyers are served. However, the seller, who sets the price after observing the segment, may choose to deviate and charge the inefficient price v_2, excluding the low-valuation buyers. An efficient equilibrium requires that the seller prefers pricing at v_1 over deviating to v_2. This condition is formalized as the seller's *Incentive Compatibility (IC)* constraint: the seller's profit from charging v_1 to the entire pooled segment must be at least as high as the profit from charging v_2 to only the v_2 buyers.

The platform's choice of q affects whether this constraint holds through two channels, both increasing the seller's incentive to deviate. First, the *direct allocation effect*: as q decreases, a larger share $1 - q$ of high-valuation v_2 buyers are sent to the pooled segment. Second, the *indirect selection effect*: a lower q increases the expected surplus for v_2 buyers,

$\kappa(\sigma|v_2) = (1-q)(v_2 - v_1)$, which increases their participation rate, $e(v_2)$ relative to the fixed captive rate $e(v_1) = \lambda$ of low-valuation buyers. Both effects increase the share of v_2 buyers in the pooled segment, strengthening the seller's temptation to charge v_2.

Formally, the seller chooses the efficient price v_1 in the pooled segment if:

$$v_1 \mu_1^0 \lambda \geq (v_2 - v_1)\big[(1-q)\mu_2^0 G\left((1-q)(v_2 - v_1)\right)\big] \tag{1}$$

The left-hand side represents the profit from the captive low-valuation buyers, which is the benefit of setting the efficient price and is constant in q. The right-hand side represents the additional profit gained from the high-valuation buyers by deviating to price v_2, which is the temptation to deviate, and is strictly decreasing in q.

Lemma 3. *There is a unique threshold $\bar{q}(\lambda) \in [0,1]$ where the IC constraint (1) binds. Efficient equilibria are sustained when*

$$q \in [\bar{q}(\lambda), 1]$$

. This implies a unique maximum implementable expected buyer surplus $\bar{\kappa} = (1-\bar{q}(\lambda))(v_2 - v_1)$, and the set of implementable buyer expected surpluses is

$$\kappa \in [0, \bar{\kappa}].$$

Proof of existence and uniqueness is in the Appendix B.

To understand the impact of this constraint, compare with the full participation benchmark, where all buyers are captive, $\lambda = 1$. Then, $e(v_1) = e(v_2) = 1$.

Proposition 3. *For any $\lambda < 1$,*

$$\bar{q}(\lambda) > \bar{q}^{BBM} \quad \textit{and} \quad \bar{\kappa}(\lambda) < \bar{\kappa}^{BBM}$$

This means the constraint on q tightens when participation is endogenous. In the full-participation case, the seller's temptation to deviate comes only from the *direct allocation effect*, that is from the share $1-q$ of v_2 buyers in the pooled segment. When $\lambda < 1$, the *indirect selection effect* amplifies this by also increasing the participation rate of v_2 buyers relative to v_1 buyers. This enriches the pooled segment with high valuations beyond the direct allocation alone, requiring a higher minimum q to satisfy the IC constraint. Hence, the threshold $\bar{q}(\lambda)$ rises, and the maximum implementable surplus $\bar{\kappa}(\lambda)$ falls compared to the full-participation benchmark.

3.2.2 The equilibrium surplus split

Having established the set of implementable segmentation rules, the platform chooses q from the interval $[\bar{q}(\lambda), 1]$ to maximize realized seller profits. Seller profits $\Pi(q;\lambda)$ come from two parts: captive low-valuation buyers, with mass $\mu_1^0\,\lambda$, served at price v_1, and participating high-valuation buyers, with mass $\mu_2^0\,e(v_2,q)$, who pay an average price of $q\,v_2 + (1-q)\,v_1$. Formally, the platform wants to

$$\max_{q\in[\bar{q}(\lambda),1]} \Pi(q;\lambda) = v_1\,\mu_1^0\,\lambda + [q\,v_2 + (1-q)\,v_1]\;\mu_2^0\,e(v_2,q)$$

To solve this constrained optimization problem, I first identify the unconstrained optimum $q^{uc}(\lambda)$, the solution on the full interval $q \in [0,1]$. This interior optimum represents the pure *participation-extraction trade-off*: attracting more high-valuation buyers versus extracting greater profit per buyer. It is uniquely characterized by the first-order condition:

$$\frac{g(\kappa(q))}{G(\kappa(q))} = \frac{1}{v_2 - \kappa(q)}, \quad \text{where} \quad \kappa(q) = (1-q)(v_2 - v_1)$$

The platform's choice is

$$q^*(\lambda) = \max\{\bar{q}(\lambda), q^{uc}(\lambda)\}$$

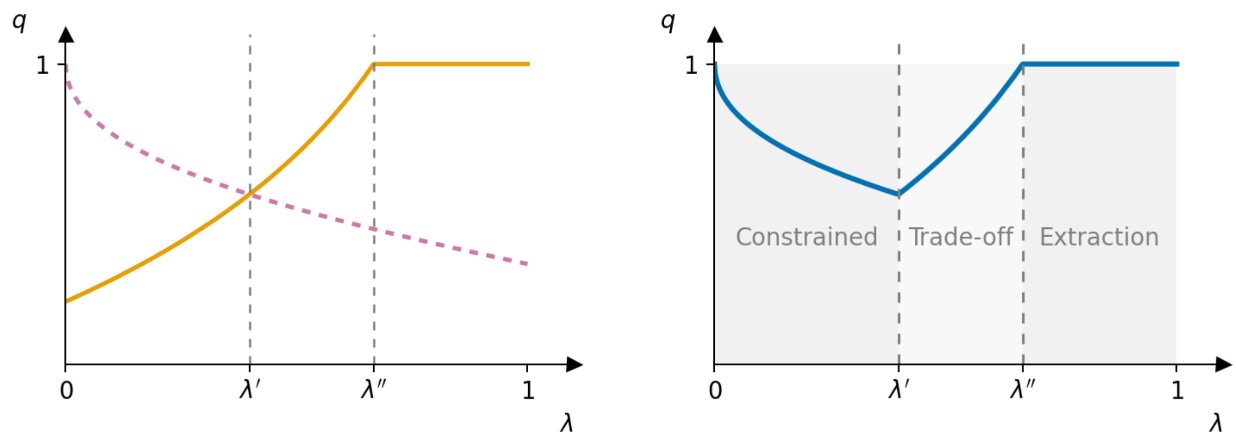
The evolution of $q^*(\lambda)$ as λ varies defines three regimes formalized in the following proposition and illustrated in Figure 4.

Proposition 4. *For thresholds* $0 < \lambda' \leq \lambda'' < 1$, *the equilibrium segmentation falls into*

1. ***Constrained regime*** *($\lambda \leq \lambda'$): The IC constraint binds,* $q^*(\lambda) = \bar{q}(\lambda)$.

2. ***Trade-off regime*** *($\lambda' < \lambda \leq \lambda''$): The IC constraint is slack,* $q^*(\lambda) = q^{uc}(\lambda)$.

3. ***Extraction regime*** *($\lambda > \lambda''$): Participation concerns vanish,* $q^*(\lambda) = 1$.



(a) Dashed $\bar{q}(\lambda)$ and solid $q^{uc}(\lambda)$.

(b) Equilibrium $q^*(\lambda) = \max\{\bar{q}(\lambda), q^{uc}(\lambda)\}$.

Figure 4: Segmentation as a function of the share of captive buyers λ. The left panel shows the two building blocks of the equilibrium, the unconstrained optimum and the constraint. The right panel displays the equilibrium segmentation $q^*(\lambda)$ and its regimes.

Two forces explain these regimes. First, the threshold of the IC constraint $\bar{q}(\lambda)$ decreases as the captive buyer share λ increases. More captive low-valuation buyers raise the seller's profit from pricing at v_1, encouraging adherence to that price and loosening the IC constraint. Second, the unconstrained optimum $q^{uc}(\lambda)$ increases with λ since a lower share of non-captive buyers decreases the platform's incentive to offer them surplus. These two functions intersect at most once, generating the three regimes. At low λ, the platform wants to offer more surplus, but is constrained; at intermediate λ, it balances extraction and participation; and at high λ, it maximizes extraction through price discrimination.

Remark 1. An increase in participation elasticity has opposite effects depending on the regime. If the optima are unconstrained, higher elasticity forces the platform to give more surplus to the buyers (lower $q^\star$). If the optima are in the constrained regime, higher elasticity tightens the IC constraint, forcing the platform towards more price discrimination (higher $q^\star$). Elasticity thus moves the solution in opposite directions across regimes and can trigger regime changes. Notably, for low λ, higher participation elasticity leads to strictly lower average buyer surplus.

The fully-informed buyer case highlights a core challenge. Buyer surplus is concentrated entirely among high-valuation buyers, who participate more than the low-valuation buyers. This self-selection reduces the share of low-valuation buyers available to pool with high-valuation ones. As a result, the platform's ability to create buyer surplus is limited by the seller's incentive compatibility constraint, which restricts the maximum share of high-valuation buyers that can be pooled. As participation becomes more elastic, this constraint tightens, lowering the maximum achievable buyer surplus and forcing the platform to enable more perfect price discrimination. Although endogenous participation encourages the platform to implement more generous surplus splits, the differential entry of buyer valuations creates an adverse selection which limits its ability to do so. In the fully-informed case, this constraint can dominate, shifting the equilibrium segmentation towards more perfect price discrimination.

4 Imperfectly informed buyers

This section analyzes the platform's problem when buyers have partial information about their valuations, represented by a belief structure β. Partially informed buyers evaluate expected surpluses $\kappa(\sigma|\beta)$ specific to their beliefs. Since these surpluses vary across different beliefs, participation decisions are correlated with valuations, resulting in *differential participation*. This process affects not only the size of the market but also its composition, making buyer participation an informative signal to the seller, though less so than in the fully informed case. To build a tractable framework that preserves the core economic trade-offs, the analysis introduces a simplifying assumption. First, the buyer population is divided into two belief groups, β^L and β^H, both belonging to $\Delta(V)$. Second, these groups satisfy the *bottom-weight(BW)* assumption: they differ only in the likelihood of being at the lowest valuation v_1, which receives zero surplus under any segmentation, while their relative beliefs over all other valuations mirror those of the initial population distribution μ^0. Formally, this is expressed as:

$$\frac{\beta_i^H}{\beta_j^H} = \frac{\beta_i^L}{\beta_j^L} = \frac{\mu_i^0}{\mu_j^0}, \quad \forall i, j \neq 1$$

The low-belief group, β^L, assigns a strictly higher probability to v_1 than β^H, that is, $\beta_1^L > \beta_1^H$. I impose lower bound on belief precision, formalized as $p^\star(\beta^L) > v_1$.

This BW assumption offers a useful and tractable way to study the problem. Because the lowest valuation, v_1, always recieves zero surplus, the differences in beliefs are confined to the state where the buyer earns zero surplus. This structure ensures that any segmentation rule σ that generates an average buyer surplus of $\kappa(\sigma)$ produces consistent surplus profiles across belief groups. Specifically, the expected surplus of a buyer with belief β is directly proportional to the aggregate average surplus $\kappa(\sigma)$ generated by that rule:

$$\kappa(\sigma|\beta) = C_\beta\, \kappa(\sigma), \quad \text{where } C_\beta = \frac{1-\beta_1}{1-\mu_1^0} \tag{2}$$

This linear relationship reduces the platform's high-dimensional problem of choosing a surplus distribution across all belief types to a one-dimensional choice over the aggregate average surplus $\kappa(\sigma)$.

4.1 The constrained buyer-optimal segmentation

To characterize the set of implementable welfare splits, a natural starting point is to identify the buyer-optimal segmentation—the rule that maximizes the surplus allocated to buyers.

4.1.1 Failure of the full-participation buyer-optimal segmentation

The natural candidate for this rule is the buyer-optimal segmentation developed in the full-participation benchmark[12], which we denote as σ^{BO}. This segmentation is constructed to maximize *pooling externalities*, where low-valuation buyers induce the seller to set lower prices in every segment, thereby generating surplus for high-valuation buyers.

The construction of σ^{BO} is as follows. First, all buyers with the lowest valuation v_1, are placed into a segment intended to be priced at v_1. To this segment, the platform adds a fraction γ_1, of buyers with higher valuations ($v > v_1$), ensuring their relative proportions from the initial market μ^0, are preserved. This fraction is calibrated to make the seller exactly indifferent between charging the lowest price v_1 and deviating to the higher uniform price p^0. The same logic is applied to create other segments for prices between v_2 and p^0, with each pooling fraction γ_k chosen to maintain seller indifference.

Under full participation, this segmentation is efficient and holds the seller to her reservation profits, π^0. This, in turn, maximizes buyer surplus at $w^0 - \pi^0$. However, as the following

[12] Section II. D of Bergemann et al. (2015)

lemma shows, this segmentation fails to achieve its intended outcome once participation becomes endogenous.

Lemma 4. *Under BW beliefs, the segmentation σ^{BO} induces no segment priced at v_1, making it inefficient.*

A full proof of this result is included in Appendix C. The intuition is as follows. Under any segmentation rule σ, buyers from the low-belief group, β^L, who are more likely to have valuation v_1, participate less often. This differential participation changes the composition of the participating market, reducing the share of v_1 relative to higher valuations.

The σ^{BO} segmentation is calibrated to the initial market, pooling buyers with valuations $v > v_1$ just to the point of making the seller indifferent. But under endogenous participation, the market becomes relatively poorer in v_1, and this balance is broken. The seller observes a pooled segment now richer in higher valuations, and has a strict incentive to deviate and set a price $p^0 > v_1$. Consequently, the segmentation becomes inefficient, as all v_1 buyers are excluded from the market.

4.1.2 Characterization of the constrained buyer-optimal segmentation rule

Given the failure of the benchmark buyer-optimal segmentation, a new approach is needed to find the rule that maximizes buyer surplus under endogenous participation. This rule must be a fixed point: it must maximize average buyer surplus for the market composition that it endogenously creates.

To ensure this fixed point is unique and well-behaved, I introduce a regularity assumption on the distribution of outside options: that *the elasticity of participation is non-decreasing in surplus on the relevant range.* This assumption has a direct implication for differential participation. Because high-belief buyers expect more surplus than low-belief buyers this assumption implies their participation will also be (weakly) more elastic.

Under this assumption, the constrained buyer-optimal segmentation is characterized by two sufficient conditions. First, *Relative Proportions (RP)*, which requires that for any segment priced below the uniform price p^0, the distribution of buyer valuations strictly above the segment's price must match the relative proportions found in the initial market. Second, *Uniform Indifference (UI)*, which requires that for any segment priced below p^0, the segmentation must be constructed such that the seller is exactly indifferent between the intended segment price and deviating to the uniform price p^0. The non-decreasing elasticity assumption ensures that a the efficient segmentation satisfying these two conditions is unique.

Proposition 5. *The unique efficient segmentation satisfying (RP) and (UI)* maximizes *realized buyer surplus among all efficient segmentation rules.*

The proof in Appendix C demonstrates that this fixed-point problem has a unique solution for the average buyer surplus , and shows that the segmentation satisfying (RP) and (UI) is this solution. The logic is as follows. First, the BW beliefs assumption ensures that the relative proportions of all valuations above v_1. are identical across belief groups. This implies that even after differential participation, the realized market $\tilde{\mu}$ has the same relative proportions above v_1 as the initial market μ^0. As a result, the seller's optimal uniform price on the realized market remains p^0. Second, the (RP) condition ensures that this same property holds within each segment. By preserving the initial market's relative proportions for all $v > p_s$ guarantees that the seller's most profitable deviation from the intended price p_s is always to p^0. Third, the (UI) condition neutralizes this deviation. It calibrates the pooling in each segment precisely so that the seller is made indifferent between p_s and the p^0 deviation. By satisfying (RP) and (UI), the segmentation holds the seller to their reservation profit on the participating market $\pi(\tilde{\mu})$. Since the rule is efficient, all remaining surplus is given to buyers. This, by construction, is the maximum possible buyer surplus that can be generated from the realized market $\tilde{\mu}$ and thus constitutes the unique fixed point.

To demonstrate existence, I use a constructive method that corrects the full-participation rule σ^{BO}. The method starts with σ^{BO} and iteratively corrects it by removing a share $\epsilon \in [0, 1]$ of high-valuation buyers $v > v_1$ from the v_1 priced segment. These buyers are then reassigned to segments priced between v_2 and p^0 in a way that preserves the Relative Proportions (RP) condition. This means that this mass is redistributed proportionally across the higher-priced segments so that each destination segment preserves the relative composition of the initial market above its own price. Increase ϵ just enough to restore the seller's Uniform Indifference (UI) condition for the v_1-segment, which was broken by differential participation. This leads to the existence of a unique, optimal correction factor, $\epsilon^\star$.

Proposition 6. *There exists a unique correction factor $\epsilon^\star \in (0,1)$ such that the ϵ-corrected segmentation maximizes buyer surplus.*

The proof in Appendix C shows that this corrected rule exists and is unique. The intuition relies on two reinforcing effects that, as ϵ increases, combine to restore the seller's incentive to price at v_1. First, the *direct allocation* effect. Increasing ϵ directly removes high-valuation buyers from the v_1-segment. This mechanically makes the segment poorer in $v > v_1$ buyers, which reduces the seller's temptation to price at p^0. Second, the *indirect selection* effect. Reallocating these buyers to higher-priced segments lowers the average expected surplus for all $v > v_1$ buyers, reducing their participation. The non-decreasing elasticity assumption ensures this participation drop is (weakly) stronger for the high-belief group. This shifts the realized market composition back towards low-valuation types, which further reinforces the seller's incentive to price at v_1.

Remark 2. The non-decreasing participation elasticity assumption ensures uniqueness of this solution. Without it, the indirect selection effect could be reversed: a decrease in expected surplus (from an increasing ϵ) might cause the participation of the low-belief group to fall *more* than that of the high-belief group. This would perversely enrich the v_1-segment with high-valuation types, further *strengthening* the seller's temptation to deviate. Such non-monotonicity could result in a set of multiple, distinct correction factors, $\Sigma = \{\epsilon \in [0,1] \mid \sigma^\epsilon \text{ satisfies (UI)}\}$, that all satisfy the seller's indifference condition. Since a larger correction ϵ implies less pooling on the lowest price segment, it unambiguously reduces buyer surplus. Therefore, the surplus-maximizing rule $\epsilon^\star$ is the the one that applies the *minimal* correction. The buyer-optimal segmentation is the one that selects $\epsilon^\star = \min \Sigma$. The non-decreasing elasticity assumption simplifies the problem by ensuring the underlying fixed-point mapping is monotonic, guaranteeing that the set Σ is a singleton and the solution is unique.

Ultimately, this construction demonstrates that while a buyer-optimal segmentation is achievable, it is fundamentally constrained by endogenous participation. The maximum average buyer surplus under this corrected rule $\kappa(\sigma^{\epsilon-BO})$, is lower than what is achievable under full participation.

4.2 The constrained welfare frontier

Having established the maximum achievable average buyer surplus, $\kappa^{\epsilon^\star}$, we can now characterize the entire set of implementable welfare outcomes. Any implementable average surplus level κ must lie in the interval $[0, \kappa^{\epsilon^\star}]$. The result is that this entire interval is implementable. Any surplus level $\kappa \in [0, \kappa^{\epsilon^\star}]$ can be achieved by a a convex combination of the seller-optimal and buyer-optimal rules: perfect price discrimination σ^{PD}, which yields $\kappa = 0$, and the constrained buyer-optimal rule $\sigma^{\epsilon^\star}$, which yields $\kappa = \kappa^{\epsilon^\star}$.

Proposition 7. *Under BW beliefs, the set of all implementable efficient average buyer surpluses κ is the interval $\kappa \in [0, \kappa^{\epsilon^\star}]$. Moreover, for every κ in this interval, there exists an efficient segmentation $\sigma_\alpha = \alpha\, \sigma^{PD} + (1-\alpha)\sigma^{\epsilon^\star}$ with $\alpha \in [0, 1]$, that achieves it.*

The proof, detailed in Appendix C, must establish that this σ_α segmentation is a valid, efficient equilibrium for all $\alpha \in [0, 1]$. The main challenge is to show that the seller's pricing incentives are unchanged in every pooled segment for all α. A failure at some intermediate α would create a hole in the frontier, making that level of surplus non-implementable.

Such a hole could plausibly form. As α increases, expected surplus κ falls. This reduces participation. If the low-belief group were to drop out faster than the high-belief group, the v_1-segment from the $\sigma^{\epsilon^\star}$ component would become richer in high-valuation types. This could break the seller's incentive constraint and induce a segment priced at $p_s < p^0$ getting priced at p^0.

This is precisely the scenario the non-decreasing participation elasticity assumption rules out. As the proof shows, this assumption implies that as average surplus κ falls, the participation of the high-belief group decreases (weakly) faster than the participation of the low-belief group.

This ensures that as α increases, the v_1-segment only becomes (weakly) less tempting for the seller to deviate from. The IC constraint therefore holds for all $\alpha \in [0, 1]$, the frontier has no holes, and the entire interval is implementable. Finally, since the average buyer surplus $\kappa(\alpha) = (1 - \alpha)\kappa^{\epsilon^\star}$ varies continuously with α, this construction spans the entire interval $[0, \kappa^{\epsilon^\star}]$.

Comparative statics. The analysis concludes by examining how the welfare frontier evolves with the model's parameters. Two factors are important: the accuracy of buyers' beliefs and the elasticity of their participation decisions. The following proposition shows that improvements in either of these dimensions shrink the set of implementable buyer-friendly outcomes.

Proposition 8. *When buyers have more precise beliefs or when their participation becomes more elastic with respect to surplus, in the sense of the reverse hazard order rate, the required segmentation correction $\epsilon^\star$ increases, and the maximum implementable buyer surplus $\kappa^{\epsilon^\star}$ decreases.*

The formal proofs are in Apendix C, but the intuition for both results comes from the same mechanism: both parameters exacerbate differential participation.

When buyers are better informed, the gap in expected surplus between the high-belief and low-belief groups widens, causing high-belief buyers to participate at a much higher relative rate.

When participation is more elastic, a given surplus difference between the two groups translates into a larger difference in their participation rates.

This intuitive story is underpinned by our standing regularity assumption. The non-decreasing participation elasticity ensures that the seller's incentive to deviate is well-behaved and monotonic in the correction factor ϵ. This regularity, which was also necessary to guarantee a unique solution in Proposition 6, is what allows for a clean comparative static. It rules out non-monotonicities that could otherwise cause $\epsilon^{\star}$ to jump discontinuously in response to these parameter changes.

With this foundation, the rest of the intuition follows directly. In both scenarios, the participating market becomes relatively richer in high-valuation buyers. This enriches the composition of the v_1 segment, strengthening the seller's incentive to deviate from the v_1 price. This tightens the seller's incentive compatibility constraint.

To counteract this and ensure the seller remains indifferent, the platform is forced to design a less generous segmentation. It must apply a larger correction factor $\epsilon^{\star}$, meaning it must reduce pooling by removing more high-valuation buyers from the v_1-priced segment. The direct consequence of this adjustment is a reduction in the maximum achievable buyer surplus $\kappa^{\epsilon^{\star}}$.

4.3 The platform's optimal segmentation

With the welfare frontier characterized, the platform's problem is to choose an average surplus κ from the feasible interval $[0, \kappa^{\epsilon^{\star}}]$ to maximize realized seller profits.

Under the BW assumption, this simplifies to a one-dimensional optimization problem:

$$\max_{\kappa \in [0, \kappa^{\epsilon^{\star}}]} \sum_{\beta} P(\beta), G(C_{\beta}\kappa), (\mathbb{E}[v|\beta] - C_{\beta}, \kappa)$$

Since the objective function is continuous and the feasible set is compact, a maximizer exists. By Proposition 7, any such maximizer is implementable by an efficient segmentation.

Comparative statics. We now examine how the platform's unconstrained optimum responds to a more elastic participation structure. Let G_1 be an initial outside-option distribution, and G_2 be a second distribution that dominates G_1 in the reverse hazard rate order. While reverse hazard rate dominance captures the direct incentive to offer more surplus, the shift from G_1 to G_2 also alters the composition of marginal entrants across belief types. This compositional effect is not governed by reverse hazard rate dominance alone. To ensure the unconstrained optimum moves monotonically, impose a mild regularity condition on the gap $H(x) := G_2(x) - G_1(x)$

Proposition 9. *Let $\overline{\kappa}^{uc}(G)$ denote the largest unconstrained maximizer of $\Pi(\kappa; G)4$. If G_2 reverse hazard rate dominates G_1 and H has non-decreasing elasticity, then*

$$\overline{\kappa}^{uc}(G_2) \geq \overline{\kappa}^{uc}(G_1)$$

In particular, if the maximizer is unique, the unconstrained maximizer $\kappa^{uc}(G)$ is non–decreasing with respect to participation elasticity.

Proof can be found in Appendix C. This proposition confirms that under this regularity condition, the pure participation-extraction trade-off still pushes the platform toward more buyer-friendly outcomes when participation becomes more elastic.

4.4 Discussion

The analysis in this paper leads to the following applied finding: forces generally considered pro-consumer, such as better buyer information or more elastic participation, have an ambiguous effect on buyer surplus. Figure 5 provides the visual summary of this tension. The equilibrium is determined by a tug-of-war between two objects that are pulled in opposite directions by these forces.

On one hand, the participation-extraction trade-off, formalized in Proposition 9, pushes the unconstrained optimum κ^{uc} to the right. A more elastic market increases the platform's

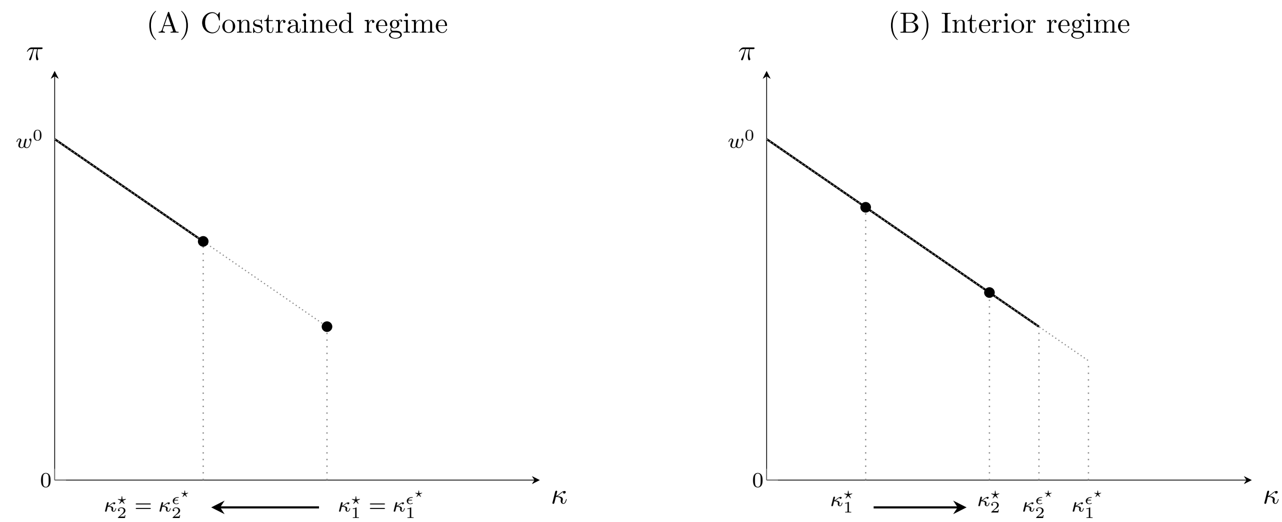


Figure 5: The ambiguous effect of participation elasticity.
Both panels illustrate the equilibrium buyer surplus $\kappa^\star$ under G_2 (more elastic) than G_1 (less elastic). Panel (A) represents the constrained regime. The equilibrium is bound by the implementability constraint. As elasticity increases, buyer surplus *decreases* ($\kappa_2^\star < \kappa_1^\star$). Panel (B) represents the unconstrained regime. The equilibrium is determined by the unconstrained trade-off. As elasticity increases, buyer surplus *increases* ($\kappa_2^\star > \kappa_1^\star$).

marginal benefit of attracting buyers, incentivizing it to offer more surplus. On the other hand, the implementability constraint, formalized in Proposition 8, tightens, pulling the buyer-optimal surplus level $\kappa^{\epsilon^\star}$ to the left.

The equilibrium depends on which of these two effects binds first. Panel (B) of the figure shows the *interior regime*, where the trade-off effect dominates and higher elasticity increases average buyer surplus. Panel (A) shows the *constrained regime*, where the implementability constraint binds. Here, the second effect dominates, and higher elasticity unambiguously decreases buyer surplus.

The economic intuition for this conflict stems from *adverse selection*. Buyer surplus is generated via pooling externalities: low-valuation buyers are pooled in segments with high-valuation buyers, which induces the seller to set a low price that benefits everyone in the pool. Differential participation directly attacks this mechanism. It is a form of adverse selection because the very buyers needed to create the externality, the low-valuation v_1 types, are also the ones who expect the least surplus and thus participate the least. Pro-consumer forces like better information or higher elasticity exacerbate this self-selection. They widen the participation gap between the high-surplus group and the low-surplus group. This starves the realized market of the low-valuation buyers required for pooling, making the market adversely selected toward high-valuation types. This, in turn, strengthens the seller's incentive to deviate to a high price, which breaks the pooling mechanism and shrinks the feasible set of buyer-friendly outcomes.

This ambiguity offers a cautionary tale. Market interventions that, on their face, appear pro-consumer, for example, policies that increase buyers' awareness of privacy costs, could have the perverse effect of reducing buyer surplus. If the market is in the constrained regime (Panel A), such an intervention would worsen adverse selection, tighten the implementability constraint, and force the platform to adopt a more extractive, less-pooling segmentation rule, harming the very buyers it was intended to help.

5 Conclusion

This paper studies a platform's optimal decision on how much price discrimination to enable when buyers can vote with their feet. This problem creates a core tension between two competing forces. The first is the participation-extraction trade-off that araises from incorporating buyer participation decisions. Providing sellers with finer buyer segments increases profits per participant but reduces market size by lowering buyers' expected surplus. The second is an implementability constraint. Because buyers' participation decisions can be correlated with their valuations, the market's composition endogenously changes. This self-selection can create a market adversely selected toward high-valuation buyers, which makes the most buyer-friendly segmentation rules, which rely on pooling, impossible to sustain in equilibrium.

The paper's first contribution is to characterize this feasible welfare frontier, showing how buyer self-selection shrinks the set of implementable outcomes. The second contribution is to show that this new constraint creates a core ambiguity in the welfare effects of pro-consumer forces. Higher participation elasticity, for instance, creates a tug-of-war of opposing effects. On one hand, it relaxes the trade-off, incentivizing the platform to offer more surplus. On the other hand, it tightens the implementability constraint by worsening adverse selection. This can paradoxically harm buyers. If the constraint binds, this force limits the platform's ability to create surplus through pooling and can forces it to default to more extractive price discrimination.

The model also opens several other promising extensions. This paper highlights that buyer information about their valuation is a key driver of the adverse selection problem. A natural next step would be to study the platform's joint design of seller segmentations and buyer-side information provision; for example, via recommendations or advertising. Furthermore, the model could be extended to consider competition between platforms, where segmentation rules and the welfare splits they imply could become a dimension of differentiation.

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A Proofs of Section 2: Model

Proof of Lemma 1. Fix σ and consider any pair (v, p) with $p > v$ and $\sigma(p \mid v) = \delta > 0$. Construct $\hat{\sigma}$ by reallocating this mass to price v: set $\hat{\sigma}(p \mid v) = 0$ and $\hat{\sigma}(v \mid v) = \sigma(v \mid v) + \delta$, leaving all other assignments unchanged. Let $\varepsilon = \delta\, \tilde{\mu}_v$ denote the post-participation mass of valuation-v buyers moved from segment p to v under $\hat{\sigma}$.

Prices. Under directness, each label equals its equilibrium price, so for all p, p',

$$p \sum_{v_k \geq p} \tilde{\mu}_k^p \;\geq\; p' \sum_{v_k \geq p'} \tilde{\mu}_k^p. \tag{IC}$$

Removing valuation-v mass from any $p > v$ leaves demand unchanged for $p' > v$ and lowers it for $p' < v$, preserving (IC). Hence $p = p^\star(\tilde{\mu}^p)$. Adding the same mass to the v-segment leaves higher-price demands unchanged and increases lower-price demands by ε, so any deviation yields a profit change $p'\varepsilon < v\varepsilon$. Thus $v = p^\star(\tilde{\mu}^v)$. All other segments are unaffected, so equilibrium prices remain unchanged.

Participation. Buyers' expected surplus depends only on the price they face when served. The reassignment converts some non-served (v, p) buyers into indifferent (v, v) buyers with zero surplus, leaving all others unchanged. Hence $e(\theta, \hat{\sigma}) = e(\theta, \sigma)$ for all θ.

Profits. Segments with $p > v$ lose only non-buyers, so their revenues are unchanged. In the v-segment, any reassigned valuation-v mass that now purchases at price v adds revenue εv. Therefore, total profits weakly increase while participation and prices remain fixed. □

B Proofs of section 3: Trade-off and constraint

B.1 Section 3.1: Uninformed buyers

Proof of Lemma 2. Let $\beta = \mu^0$ and $\kappa(\sigma) \equiv \kappa(\sigma \mid \mu^0)$; set $e := G(\kappa(\sigma))$.

Segment compositions. Participation is proportional, so $\tilde{\mu} = e\,\mu^0$; applying σ preserves within–segment shares, hence $p_s = p^\star(\mu^s)$ for every segment.

Per–participant equivalence. $K(\sigma) = e\,\kappa(\sigma)$, so per–participant buyer surplus is $\kappa(\sigma)$. Since σ is efficient, all participants are served; proportional participation keeps the average valuation at w^0, hence per–participant seller profit is $w^0 - \kappa(\sigma)$.

Feasible interval. By BBM Theorem 1, efficient segmentations under full participation implement $\kappa \in [0, w^0 - \pi^0]$. Because segment compositions and per–participant welfare coincide with the full-participation benchmark here, the same interval applies. □

Proof of Proposition 1. The platform chooses $\kappa \in [0, w^0 - \pi^0]$ to maximize $\Pi(\kappa) = G(\kappa)(w^0 - \kappa)$. Since G is log–concave and $w^0 - \kappa$ is linear and positive on the interior, Π is strictly log–concave on $(0, w^0 - \pi^0)$; on a compact interval this yields existence and uniqueness of the maximizer $\kappa^\star$. Moreover, $\Pi(0) = G(0)w^0 = 0$ and $\Pi(w^0 - \pi^0) = G(w^0 - \pi^0)(w^0 - \pi^0) > 0$, so $\kappa^\star \in (0, w^0 - \pi^0]$.

If the maximizer is interior, the FOC is

$$\Pi'(\kappa^\star) = g(\kappa^\star)(w^0 - \kappa^\star) - G(\kappa^\star) = 0 \quad \Longleftrightarrow \quad \frac{g(\kappa^\star)}{G(\kappa^\star)} = \frac{1}{w^0 - \kappa^\star}.$$

If no interior solution exists on $(0, w^0 - \pi^0)$, the unique maximizer is the upper corner $\kappa^\star = w^0 - \pi^0$. □

B.2 Proofs of section 3.2: Fully-informed buyers

Proof of Lemma 3. Define $F(q) := (v_2 - v_1)(1-q)\,\mu_2^0\,G\big((1-q)(v_2 - v_1)\big) - v_1\mu_1^0\lambda$.

Continuity. G is continuous, hence F is continuous on $[0, 1]$.

Strict monotonicity. Let $x := (1-q)(v_2 - v_1)$. Then

$$F(q) = (v_2 - v_1)\mu_2^0\,(1-q)G(x) - v_1\mu_1^0\lambda, \quad F'(q) = -(v_2 - v_1)\mu_2^0\big[G(x) + (1-q)(v_2 - v_1)g(x)\big] < 0,$$

since $G(x) > 0$, $g(x) \geq 0$, and $v_2 > v_1$. Thus F is strictly decreasing on $[0, 1]$.

Endpoints. At $q = 1$, $F(1) = -v_1\mu_1^0\lambda < 0$. At $q = 0$,

$$F(0) = (v_2 - v_1)\mu_2^0\, G(v_2 - v_1) - v_1\mu_1^0\lambda > 0,$$

because $G(v_2 - v_1) > \lambda$, and $(v_2 - v_1)\mu_2^0 > v_1\mu_1^0$ by $p_0 = v_2$ in the initial market.

Existence and uniqueness. A continuous, strictly decreasing F with $F(0) > 0$ and $F(1) < 0$ has a unique root $\bar{q}(\lambda) \in (0, 1)$ by the Intermediate Value Theorem.

Proof of Proposition 3. In any pooled segment with composition $\mu \in \Delta(\{v_1, v_2\})$, the seller's IC binds at

$$v_1\mu_1 = (v_2 - v_1)\mu_2 \quad \Longleftrightarrow \quad R(q) := \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2 - v_1}.$$

Under full participation,

$$R^{BBM}(q) = \frac{\mu_2^0(1 - q)}{\mu_1^0}.$$

With endogenous participation,

$$R(q) = \frac{\mu_2^0(1 - q)\, G\big((1 - q)(v_2 - v_1)\big)}{\mu_1^0\, \lambda} = R^{BBM}(q) \cdot \frac{G((1 - q)(v_2 - v_1))}{\lambda}.$$

Since $G(x) \geq \lambda$ for $x \geq 0$, we have $R(q) \geq R^{BBM}(q)$. Thus, for the common IC threshold $y = \frac{v_1}{v_2 - v_1}$, the strictly decreasing curves R^{BBM} and R cross it at $\bar{q}^{BBM} \leq \bar{q}$.

Proof of Proposition 4. *Roadmap.* We show (a) $\bar{q}'(\lambda) < 0$, the IC threshold decreases with λ, (b) $(q^{uc})'(\lambda) > 0$ the unconstrained optimum increases with λ, (c) single crossing, and (d) boundary values pinning the three regimes.

(a) $\bar{q}'(\lambda) < 0$. Let

$$F(q,\lambda) := (v_2 - v_1)(1-q)\mu_2^0\, G\big(\kappa(q)\big) - v_1\mu_1^0\lambda,$$

so $\bar{q}(\lambda)$ solves $F(\bar{q},\lambda) = 0$. By Lemma 3, $F_q < 0$. Decompose $G(x) = \lambda + (1-\lambda)\tilde{G}(x)$; then

$$F_\lambda(\bar{q},\lambda) = (v_2 - v_1)(1-\bar{q})\mu_2^0\big(1 - \tilde{G}\big) - v_1\mu_1^0.$$

Using $F(\bar{q},\lambda) = 0$ to substitute $v_1\mu_1^0 = (v_2 - v_1)(1-\bar{q})\mu_2^0\, G/\lambda$ gives

$$F_\lambda(\bar{q},\lambda) = -(v_2 - v_1)(1-\bar{q})\mu_2^0\,\frac{\tilde{G}}{\lambda} < 0.$$

By the IFT, $\bar{q}'(\lambda) = -F_\lambda/F_q < 0$.

(b) $(q^{uc})'(\lambda) > 0$. Define

$$H(q,\lambda) := \frac{g(\kappa(q))}{G(\kappa(q))} - \frac{1}{v_2 - \kappa(q)}.$$

The unconstrained maximizer q^{uc} solves $H(q^{uc},\lambda) = 0$. Differentiating in q,

$$H_q = \kappa'(q)\left(\frac{\mathrm{d}}{\mathrm{d}\kappa}\frac{g}{G} - \frac{\mathrm{d}}{\mathrm{d}\kappa}\frac{1}{v_2 - \kappa}\right) > 0,$$

since $\kappa'(q) = -(v_2 - v_1) < 0$, $(g/G)' < 0$, and $(1/(v_2-\kappa))' > 0$. Differentiating in λ via $G(\kappa) = \lambda + (1-\lambda)\tilde{G}(\kappa)$ yields

$$H_\lambda = -\frac{\tilde{g}(\kappa)}{\big(\lambda + (1-\lambda)\tilde{G}(\kappa)\big)^2} < 0.$$

IFT gives $(q^{uc})'(\lambda) = -H_\lambda/H_q > 0$.

(c) Single crossing. Since $\bar{q}(\lambda)$ decreases and $q^{uc}(\lambda)$ increases, their graphs intersect at most once.

(d) Boundary values and regimes. As $\lambda \to 0$, the IC equation

$$(v_2 - v_1)(1 - \bar{q})\mu_2^0 G(\kappa) = v_1 \mu_1^0 \lambda$$

forces $(1 - \bar{q})\tilde{G}(\kappa) = 0$, hence $\lim_{\lambda \to 0} \bar{q}(\lambda) = 1$. For q^{uc}, the FOC is $\tilde{g}(\kappa)/\tilde{G}(\kappa) = 1/(v_2 - \kappa)$; since $\tilde{g}/\tilde{G} \to \infty$ as $\kappa \to 0$, we must have $\lim_{\lambda \to 0} q^{uc} < 1$. At $\lambda = 1$, full participation gives $q^{uc} = 1$, while the IC threshold solves $(v_2 - v_1)(1 - \bar{q})\mu_2^0 = v_1 \mu_1^0$, so $\bar{q} < 1$. By continuity there is a unique $\lambda \in (0, 1)$ with $\bar{q}(\lambda) = q^{uc}(\lambda)$, which yields the three regimes. □

C Proofs of section 4: Imperfectly informed buyers

Standing notation and preliminary results for Section 4

Denote as

$$L(\kappa) := \sum_{\beta} P(\beta)\, G(C_\beta\, \kappa)\, \beta_1, \qquad S(\kappa) := \sum_{\beta} P(\beta)\, G(C_\beta\, \kappa)\, (1 - \beta_1), \tag{3}$$

$$H(\kappa) := \frac{L(\kappa)}{S(\kappa)}. \tag{4}$$

Meaning. $L(\kappa)$ and $S(\kappa)$ are, respectively, the participating masses of v_1 buyers and of $v > v_1$ buyers; $H = L/S$ is their ratio.

Lemma 5. *For any efficient segmentation σ and any $k > 1$, the realized mass satisfies $\tilde{\mu}_k = c\, \mu_k^0$ for some $c > 0$ (independent of k).*

Proof. Under BW, for any belief β and any $i, j > 1$, $\beta_i = \mu_i^0\, \beta_j / \mu_j^0$, hence $\beta_j = \frac{1-\beta_1}{1-\mu_1^0}\, \mu_j^0$. Therefore, for $i > 1$,

$$\tilde{\mu}_i = \sum_{\beta} P(\beta)\, G(C_\beta \kappa)\, \beta_i = \mu_i^0 \sum_{\beta} P(\beta)\, G(C_\beta \kappa) \frac{1 - \beta_1}{1 - \mu_1^0} =: c\, \mu_i^0,$$

with $c > 0$ common to all $i > 1$. □

Lemma 6. *Fix a market μ with uniform monopoly price $p^\dagger := p^\star(\mu)$ and a floor p. If another market μ' satisfies $\mu'_k = c\,\mu_k$ for all $v_k > p$ (some $c > 0$), then*

$$\arg\max_{r \ge p}\; r \sum_{v_k \ge r} \mu'_k \;=\; \arg\max_{r \ge p}\; r \sum_{v_k \ge r} \mu_k.$$

In particular, if $p \le p^\dagger$, the seller's best deviation within $\{r \ge p\}$ is $p^\dagger$ (weakly unique under the lowest-price tie-break).

Proof. For any $r \ge p$, $\sum_{v_k \ge r} \mu'_k = c \sum_{v_k \ge r} \mu_k$, hence $r \sum_{v_k \ge r} \mu'_k = c \cdot r \sum_{v_k \ge r} \mu_k$. Proportional objective functions have identical maximizers; the tie-break carries over. □

Corollary 1. *In the realized market $\tilde{\mu}$, the uniform monopoly price is $p^\star(\tilde{\mu}) = p^0$.*

Proof. By Lemma 5, for every $r > v_1$ the tail masses satisfy $\sum_{v_k \ge r} \tilde{\mu}_k = c \sum_{v_k \ge r} \mu^0_k$ with $c > 0$. Since $p^0 > v_1$ (by definition of p^0 in the initial market), apply Lemma 6 to the pair $(\mu^0, \tilde{\mu})$ with floor $p = v_1$ to get that the maximizer over all $r > v_1$ is unchanged: $p^\star(\tilde{\mu}) = p^\star(\mu^0) = p^0$. □

Corollary 2. *Consider any segment s intended at price p that* preserves relative proportions for all $v > v_1$*: there exists $c_s > 0$ such that $\tilde{\mu}^s_k = c_s\,\tilde{\mu}_k$ for every $v_k > v_1$. Then the seller's most profitable deviation within segment s is p^0.*

Proof. By the proportionality assumption, for any $r \ge p$, $r \sum_{v_k \ge r} \tilde{\mu}^s_k = c_s\, r \sum_{v_k \ge r} \tilde{\mu}_k$. By Lemma 6 with baseline $\tilde{\mu}$ and floor p, the argmax over $\{r \ge p\}$ is the same as in $\tilde{\mu}$, which is $p^\star(\tilde{\mu}) = p^0$ by Corollary 1. □

Lemma 7. *If the participation elasticity $\varepsilon_G(x) := \frac{x\,g(x)}{G(x)}$ is non-decreasing in x on the relevant range, then $H(\kappa) = L(\kappa)/S(\kappa)$ is strictly decreasing in κ.*

Proof. Compute log-derivatives:

$$\frac{L'}{L} = \sum_\beta \underbrace{\frac{P(\beta)\,\beta_1\,G(C_\beta\kappa)}{L(\kappa)}}_{:=\rho^L_\beta} \cdot \frac{\varepsilon_G(C_\beta\kappa)}{\kappa}, \qquad \frac{S'}{S} = \sum_\beta \underbrace{\frac{P(\beta)\,(1-\beta_1)\,G(C_\beta\kappa)}{S(\kappa)}}_{:=\rho^S_\beta} \cdot \frac{\varepsilon_G(C_\beta\kappa)}{\kappa}.$$

Under BW, C_β is decreasing in β_1, and by assumption $\varepsilon_G(C_\beta \kappa)/\kappa$ is therefore decreasing in β_1. The weights $\rho^S_\beta/\rho^L_\beta$ are decreasing in β_1, so by a Chebyshev/monotone-weights argument, $\frac{S'}{S} > \frac{L'}{L}$, hence $\frac{d}{d\kappa}(L/S) < 0$. □

C.1 Section 4.1: The constrained buyer-optimal segmentation

Proof of Lemma 4. The buyer–optimal rule σ^{BO} calibrates the v_1–segment at full participation to make the seller indifferent between v_1 and p^0:

$$v_1\left(\mu^0_1 + \sum_{k=2}^{K} \mu^0_k\, \sigma(s_1 \,|\, v_k)\right) \;=\; p^0 \sum_{k:\, v_k \geq p^0} \mu^0_k\, \sigma(s_1 \,|\, v_k). \tag{5}$$

After endogenous participation, denote $\tilde{\mu}^{v_1}_k$ the realized mass of type v_k inside the v_1–segment. The seller prefers to deviate to p^0 iff

$$v_1\left(\tilde{\mu}_1 + \sum_{k=2}^{K} \tilde{\mu}_k\, \sigma(s_1 \,|\, v_k)\right) \;<\; p^0 \sum_{k:\, v_k \geq p^0} \tilde{\mu}_k\, \sigma(s_1 \,|\, v_k). \tag{6}$$

Step 1 (reduce to a single comparison). Under BW, realized masses above v_1 scale proportionally: for all $k > 1$, $\tilde{\mu}_k = c\,\mu^0_k$ with the same $c > 0$ (Lemma 5). Substituting into (6) and using (5) for the bracketed term yields

$$v_1\left(\tilde{\mu}_1 + c\sum_{k=2}^{K} \mu^0_k\, \sigma(s_1 \,|\, v_k)\right) \;<\; c\, v_1\left(\mu^0_1 + \sum_{k=2}^{K} \mu^0_k\, \sigma(s_1 \,|\, v_k)\right),$$

so, after cancelling the common sum and v_1,

$$\tilde{\mu}_1 \;<\; c\,\mu^0_1. \tag{7}$$

Thus the deviation is profitable iff the realized mass of v_1 buyers is relatively smaller than that of $v > v_1$ buyers.

Step 2 (the v_1 mass is relatively smaller under participation). By definition,

$$\tilde{\mu}_1 - c\mu_1^0 = \sum_{\beta} P(\beta)\, G(C_\beta \kappa(\sigma)) \Big(\beta_1 - \mu_1^0\Big).$$

With two BW belief groups β^L, β^H, the law of total probability gives $P(\beta^L)(\beta_1^L - \mu_1^0) = -P(\beta^H)(\beta_1^H - \mu_1^0)$, so

$$\tilde{\mu}_1 - c\,\mu_1^0 = P(\beta^H)\,(\beta_1^H - \mu_1^0)\,\Big(G(C_H\kappa(\sigma)) - G(C_L\kappa(\sigma))\Big).$$

Here $(\beta_1^H - \mu_1^0) < 0$ and $C_H > C_L$, while G is non–decreasing; if $\kappa(\sigma) > 0$, then $G(C_H\kappa(\sigma)) \geq G(C_L\kappa(\sigma))$, with strict $>$ whenever $C_H\kappa(\sigma) > C_L\kappa(\sigma)$. Hence $\tilde{\mu}_1 - c\,\mu_1^0 < 0$, i.e. (7) holds.

Conclusion. The seller strictly prefers p^0 to v_1 in the v_1–segment, so no segment can be sustained at v_1; σ^{BO} is therefore not an equilibrium under endogenous participation. □

Proof of Proposition 5. *Roadmap.* We proceed in four steps. First, we show that the realized buyer surplus K is strictly increasing in the average surplus κ, so the platform's problem reduces to choosing κ. Second, equating $K(\kappa)$ to realized welfare minus realized profits yields the equilibrium condition $\kappa = \Phi(\kappa)$. Third, we prove Φ is strictly decreasing, so $M^\star(\kappa) := \Phi(\kappa) - \kappa$ crosses zero exactly once, delivering a unique $\kappa^\star$. Fourth, we construct an efficient rule satisfying (RP) and (UI) that attains $\kappa^\star$, establishing existence, uniqueness, and attainment of the buyer–optimal equilibrium.

Step 1 (Reduction to κ). Let $f(x) := xG(x)$, so $f'(x) = G(x) + xg(x) > 0$ for $x > 0$. Then

$$K(\kappa) = \sum_{\beta} P(\beta)\, f(C_\beta \kappa) \quad \Rightarrow \quad \frac{dK}{d\kappa} = \sum_{\beta} P(\beta)\, f'(C_\beta\kappa)\, C_\beta > 0.$$

Hence maximizing K is equivalent to maximizing κ.

Step 2 (Fixed–point equation). Under BW, expected surplus for belief β equals $C_\beta \kappa$, hence

$$K(\kappa) = \kappa \, \frac{S(\kappa)}{1-\mu_1^0}.$$

Realized total welfare is

$$W(\tilde{\mu}(\kappa)) = v_1 L(\kappa) + \sum_{k>1} v_k \, \frac{S(\kappa)}{1-\mu_1^0} \, \mu_k^0,$$

and realized profits are evaluated at $p^\star(\tilde{\mu}) = p^0$ (Corollary 1), so

$$\Pi(\tilde{\mu}(\kappa)) = p^0 \sum_{k:\, v_k \geq p^0} \tilde{\mu}_k = p^0 \sum_{k:\, v_k \geq p^0} \frac{S(\kappa)}{1-\mu_1^0} \, \mu_k^0.$$

Hence

$$W(\tilde{\mu}(\kappa)) - \Pi(\tilde{\mu}(\kappa)) = v_1 L(\kappa) + S(\kappa)\, \Delta, \qquad \Delta := \frac{\sum_{k>1} v_k \mu_k^0 - p^0 \sum_{k:\, v_k \geq p^0} \mu_k^0}{1-\mu_1^0} > 0.$$

At a buyer–optimal equilibrium, $K = W - \Pi$, giving

$$\kappa \, \frac{S(\kappa)}{1-\mu_1^0} = v_1 L(\kappa) + S(\kappa)\Delta \quad \Longleftrightarrow \quad \kappa = \Phi(\kappa) := (1-\mu_1^0)\left(v_1 \frac{L(\kappa)}{S(\kappa)} + \Delta \right).$$

(Uses (3)–(4).) *Step 3 (Uniqueness of the fixed point).* By Lemma 7, $H(\kappa) = L(\kappa)/S(\kappa)$ is strictly decreasing; thus

$$\Phi'(\kappa) = (1-\mu_1^0)\, v_1 \, H'(\kappa) < 0.$$

Therefore $M^\star(\kappa) := \Phi(\kappa) - \kappa$ is strictly decreasing and continuous. For small κ, $M^\star(\kappa) \to \Phi(0) > 0$; for large enough κ, $M^\star(\kappa) < 0$. By the Intermediate Value Theorem there exists a unique $\kappa^\star$ with $\Phi(\kappa^\star) = \kappa^\star$.

Step 4 (Attainment via (RP)+(UI)). (RP) implies that, in any sub–p^0 segment, the seller's best deviation is p^0 (Lemma 6 and Corollary 1); hence deviations other than p^0 are never profitable in those segments. (UI) calibrates each such segment so that the seller is indifferent between its intended price p_s and p^0:

$$p_s \sum_{v_k \geq p_s} \tilde{\mu}_k^s = p^0 \sum_{v_k \geq p^0} \tilde{\mu}_k^s,$$

and summing across segments yields realized profits equal to uniform–price profits $p^0 \sum_{v_k \geq p^0} \tilde{\mu}_k$. Because the rule is efficient, all remaining realized welfare accrues to buyers, attaining the maximal buyer surplus compatible with $\tilde{\mu}(\kappa^\star)$. Combined with Step 3, this shows that (RP)+(UI) implements the unique fixed point $\kappa^\star$, completing the proof. □

Proof of Proposition 6. Let $F(\epsilon)$ denote the v_1–segment IC gap. In the v_1–segment we have $\tilde{\mu}_1^{v_1} = L(\epsilon)$ and, by construction of the ϵ–correction, the pooled $v > v_1$ mass equals $(1-\epsilon)\gamma_1 S(\epsilon)$ and is allocated according to the BO weights (with pooling constant γ_1). Using the BO calibration at full participation,

$$p^0 \sum_{v_k \geq p^0} \gamma_1 \mu_k^0 = v_1 \left(\mu_1^0 + \sum_{k>1} \gamma_1 \mu_k^0 \right) =: v_1\, \mu_1^0\, A \qquad \Big(A = 1 + \gamma_1 \frac{1-\mu_1^0}{\mu_1^0} > 0 \Big),$$

one obtains, after substitution and factorization,

$$F(\epsilon) \; = \; v_1 \Big(L(\epsilon) + (1-\epsilon) S(\epsilon) \big(\gamma_1 - \frac{\mu_1^0}{1-\mu_1^0}\, A \big) \Big) \; = \; v_1\, S(\epsilon) \, \Big(H(\epsilon) - \tfrac{\mu_1^0}{1-\mu_1^0} (1-\epsilon) \Big).$$

Set $J(\epsilon) := H(\epsilon) - \frac{\mu_1^0}{1-\mu_1^0}(1-\epsilon)$. Since $S(\epsilon) > 0$, the IC binds iff $J(\epsilon) = 0$.

Endpoints. At $\epsilon = 0$ the BO rule fails the IC in the v_1–segment (Lemma 4), so $J(0) < 0$. At $\epsilon = 1$, the v_1–segment contains only v_1–types, so $J(1) = H(1) > 0$. By continuity, there exists $\bar{\epsilon} \in (0,1)$ with $J(\bar{\epsilon}) = 0$.

Monotonicity. Along the correction path $\kappa(\epsilon)$ strictly decreases, hence by Lemma 7 the map $H(\epsilon) = H(\kappa(\epsilon))$ is strictly increasing. Therefore

$$J'(\epsilon) = H'(\epsilon) + \frac{\mu_1^0}{1-\mu_1^0} \;>\; 0,$$

so J is strictly increasing on $[0,1)$.

Existence and uniqueness. Since J is continuous, strictly increasing, and $J(0) < 0 < J(1)$, there exists a unique $\epsilon^\star \in (0,1)$ with $J(\epsilon^\star) = 0$, i.e.

$$\epsilon^\star \;=\; 1 - H(\epsilon^\star)\,\frac{1-\mu_1^0}{\mu_1^0}.$$

This $\epsilon^\star$ is the unique correction that restores the v_1–segment IC at equality. □

C.2 Section 4.2: The constrained welfare frontier

Proof of Proposition 7. By construction, $\sigma_\alpha := \alpha\,\sigma^{PD} + (1-\alpha)\,\sigma^{\epsilon^\star}$ is Bayes–plausible and efficient, and for any $p < p^0$ the within–p composition above p equals that under $\sigma^{\epsilon^\star}$ (since $\sigma^{PD}(p \mid v_k) = 0$ for $v_k > p$).

Under BW, participation scales all types $v > p$ in any segment by the same factor (Lemma 5), so (RP) is preserved post–participation. Hence, by Lemma 6 and Corollary 1, the seller's best deviation in any sub–p^0 segment is p^0.

Fix α.

Case $p > v_1$. Let $A > 0$ denote the post–participation mass with $v = p$ in the p–segment, and B, C the pooled masses (at $\alpha = 0$) with $v > p$ and $v \geq p^0$, respectively. Revenues satisfy

$$R_p(\alpha) = p\,[A + (1-\alpha)B], \qquad R_{p^0}(\alpha) = p^0\,(1-\alpha)C,$$

and (UI) at $\alpha = 0$ gives $p(A+B) = p^0C$. Thus

$$R_p(\alpha) - R_{p^0}(\alpha) = \alpha\,pA > 0,$$

so p strictly dominates p^0 for all $\alpha > 0$ (indifference at $\alpha = 0$). By Step 1, no other deviation is more profitable.

Case $p = v_1$. Using the binding IC at $\alpha = 0$ and the BO weights as in Prop. 6, the gap is

$$F(\alpha) = v_1 \, S(\kappa(\alpha))\Big(H(\kappa(\alpha)) - (1-\alpha)H(\kappa(0))\Big).$$

Along the α–path, $\kappa(\alpha)$ decreases and Lemma 7 implies $H(\kappa(\alpha))$ increases. Hence, for any $\alpha > 0$,

$$H(\kappa(\alpha)) - (1-\alpha)H(\kappa(0)) \;\geq\; H(\kappa(0)) - (1-\alpha)H(\kappa(0)) = \alpha\, H(\kappa(0)) > 0,$$

so $F(\alpha) > 0$ and the optimal price remains v_1.

Combining the two cases, every sub–p^0 segment is priced at its intended p, and σ_α is an equilibrium efficient segmentation for all $\alpha \in [0, 1]$.

For any belief β, $\kappa(\sigma|\beta)$ is linear in σ,

$$\kappa(\sigma_\alpha \mid \beta) = \alpha\, \kappa(\sigma^{PD} \mid \beta) + (1-\alpha)\, \kappa(\sigma^{\epsilon^\star} \mid \beta) = (1-\alpha) C_\beta\, \kappa^{\epsilon^\star},$$

so under BW, $\kappa(\alpha) = (1-\alpha)\, \kappa^{\epsilon^\star}$. Thus $\{\kappa(\alpha) : \alpha \in [0, 1]\} = [0, \kappa^{\epsilon^\star}]$.

By Proposition 5, no efficient rule can deliver $\kappa > \kappa^{\epsilon^\star}$. Hence the constrained frontier is exactly $[0, \kappa^{\epsilon^\star}]$. □

Proof of Proposition 8. By Proposition 7, $\epsilon^\star$ is the unique root of $J(\epsilon, \theta) = 0$ with $J(\epsilon, \theta) := H(\epsilon, \theta) - (1-\epsilon)\frac{\mu_1^0}{1-\mu_1^0} A$. By the Implicit Function Theorem,

$$\operatorname{sign}\!\left(\frac{d\epsilon^\star}{d\theta}\right) = -\operatorname{sign}\!\left(\frac{\partial J}{\partial \theta}(\epsilon^\star, \theta)\right) = -\operatorname{sign}\!\left(\frac{\partial H}{\partial \theta}(\epsilon^\star, \theta)\right),$$

so it suffices to show $\partial_\theta H < 0$ in each comparative-static.

Re-weighting identity. Let $e_L := G(C_L \kappa)$ and $e_H := G(C_H \kappa)$ be the participation rates of the low- and high-belief groups, and set $R := e_H/e_L$ and $\rho := P(H)/P(L =$. Using $L(\kappa) = P(L)e_L\beta_1^L + P(H)e_H\beta_1^H$ and $S(\kappa) = P(L)e_L(1-\beta_1^L) + P(H)e_H(1-\beta_1^H)$,

$$H(\kappa) = \frac{L}{S} = \frac{\beta_1^L + \rho R\,\beta_1^H}{(1-\beta_1^L) + \rho R\,(1-\beta_1^H)}. \tag{8}$$

Holding beliefs fixed, differentiation of (8) gives

$$\frac{\partial H}{\partial R} = \frac{\rho(\beta_1^H - \beta_1^L)}{\big((1-\beta_1^L) + \rho R(1-\beta_1^H)\big)^2} < 0 \quad \text{since } \beta_1^L > \beta_1^H.$$

(i) More elastic participation (RHRD). Let G_2 dominate G_1 in the reverse hazard rate order. Then for any $\kappa_H > \kappa_L$, $\frac{G_2(\kappa_H)}{G_2(\kappa_L)} > \frac{G_1(\kappa_H)}{G_1(\kappa_L)}$, so R increases with θ while beliefs are fixed; hence $\partial_\theta H = \frac{\partial H}{\partial R}\frac{\partial R}{\partial \theta} < 0$. Therefore $d\epsilon^\star/d\theta > 0$.

(ii) More precise beliefs (mean-preserving spread). A BW–precision increase raises C_H and lowers C_L, so (with G increasing) e_H rises and e_L falls, i.e. R increases; this *indirect* effect makes H fall by $\partial H/\partial R < 0$. There is also a *direct* effect via the belief parameters: using $\beta_1 = 1 - C_\beta(1-\mu_1^0)$ in (8),

$$H = \frac{1 + \rho R - (1-\mu_1^0)\big(C_L + \rho R C_H\big)}{(1-\mu_1^0)\big(C_L + \rho R C_H\big)} = \frac{1+\rho R}{(1-\mu_1^0)X} - 1, \qquad X := C_L + \rho R C_H.$$

Holding R constant, under a spread, $dX = \rho(R-1)\,dC_H > 0$ (since $R > 1$ along the BW ray when $C_H > C_L$), and $\partial H/\partial X < 0$, so this direct effect also lowers H. Thus $\partial_\theta H < 0$ and again $d\epsilon^\star/d\theta > 0$.

In both cases, $\epsilon^\star$ increases with θ. Since $\kappa(\epsilon)$ is strictly decreasing in ϵ along the correction path, the maximum implementable buyer surplus $\kappa^{\epsilon^\star} = \kappa(\epsilon^\star)$ strictly decreases with θ. □

C.3 Section 4.3: The platform's optimal segmentation

Define $H(x) = \tilde{G}(x) - G(x)$. Since $\tilde{G}$ dominates G in the reverse hazard rate order, then $\tilde{G}$ first order stochastically dominates G, which means $H(x) \leq 0$ for all x.

Denote $\Delta(\kappa) = \Pi_{\tilde{G}}(\kappa) - \Pi_G(\kappa)$. We want to show that $\Delta(\kappa)$ is single-crossing increasing. That is, for $\kappa'' \geq \kappa'$, $\Delta(\kappa') \geq 0 \implies \Delta(\kappa'') \geq 0$.

We can write $\Delta(\kappa) = \sum_\beta P(\beta)H(C_\beta\kappa)(\mathbb{E}[v|\beta] - C_\beta\kappa)$ We can re-write it as

$$\Delta(\kappa) = \Bigg(\underbrace{\frac{\sum_\beta P(\beta)H(C_\beta\kappa)\mathbb{E}[v|\beta]}{\sum_\beta P(\beta)H(C_\beta\kappa)C_\beta}}_{=:R(\kappa)} - \kappa\Bigg)\sum_\beta \underbrace{P(\beta)H(C_\beta\kappa)C_\beta}_{<0}$$

Therefore,

$$\text{sign}(\Delta) = \text{sign}(-(R(\kappa) - \kappa))$$

Define the following: $p_\beta(\kappa) = \frac{P(\beta)H(C_\beta\kappa)C_\beta}{\sum_{\beta'} P(\beta')H(C_{\beta'}\kappa)C_{\beta'}}$, where $p_\beta(\kappa) \geq 0$ and $\sum_\beta p_\beta(\kappa) = 1$

Then, we can write

$$R'(\kappa) = \sum_\beta p_\beta(\kappa)\frac{\mathbb{E}[v|\beta]}{C_\beta}C_\beta\frac{H'(C_\beta\kappa)}{H(C_\beta\kappa)} - \Big(\sum_\beta p_\beta(\kappa)\frac{\mathbb{E}[v|\beta]}{C_\beta)}\Big)\Big(\sum_\beta p_\beta(\kappa)C_\beta\frac{H'(C_\beta\kappa)}{H(C_\beta\kappa)}\Big)$$

This means $R'(\kappa) = \text{Cov}_{p(\kappa)}(\frac{\mathbb{E}[v|\beta]}{C_\beta}, C_\beta\frac{H'(C_\beta\kappa)}{H(C_\beta\kappa)})$

Where, with the BW assumption, one can find that $\frac{\mathbb{E}[v|\beta]}{C_\beta}$ is decreasing in C_β, and given that $x\frac{H'(x)}{H(x)}$ is non-decreasing in x, then $C_\beta\frac{H'(C_\beta\kappa)}{H(C_\beta\kappa)}$ is increasing in C_β. Then, $R'(\kappa) = \text{Cov}_{p(\kappa)}(\frac{\mathbb{E}[v|\beta]}{C_\beta}, C_\beta\frac{H'(C_\beta\kappa)}{H(C_\beta\kappa)}) \leq 0$

Therefore $\frac{d}{d\kappa}R(\kappa) - \kappa = R'(\kappa) - 1 \leq 0$

Now, if $\Delta(\kappa') \geq 0$, then $R(\kappa') - \kappa' \leq 0$. Consider $\kappa'' > \kappa'$. Since $R(\kappa') - \kappa'$ is decreasing, then $R(\kappa'') - \kappa'' \leq 0$. We can therefore conclude that $\Delta(\kappa'') \geq 0$. Therefore, Δ is single-crossing increasing. By Milgrom-Shannon monotone selection results, the largest maximiser $\overline{\kappa}$ is non-decreasing in the reverse-hazard rate order.

□